Blended Experimentation with DGS

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Experimentation is an important element of science teaching. Students should experience real-world phenomena, and support or disprove their hypotheses about the world by setting up appropriate situations, conducting the experiments, gathering data and drawing conclusions. Unfortunately, many experiments are unsuitable for school teaching out of various reasons. They might be too dangerous, too expensive, too complex, too unreliable, or they take too much time to complete.

Using simulation software, many experiments can be replaced by safer, cheaper, easier, more reliable or sped-up virtual counterparts. Such simulations range from showing videos of repeated runs of the real experiment, together with interaction facilities, to mathematical simulations using numerical solutions of partial differential equations.

However, there is a pedagogical drawback of the simulation approach: The connection to the real world situation is (at least partially) lost. This might harm both the motivation of the students and their belief in the conclusions.

In this paper we describe the approach of blended experimentation, where simulations and the real world are connected via sensors and actors. The sensors are able to influence the simulation, while the actors can change the real world. We show first implementations and examples using Dynamic Geometry Software and other Mathematical Software as simulation environments.

1   INTRODUCTION

There is no doubt that the introduction of computers has changed or will change teaching and learning, in particular of mathematics and science. In particular, the introduction of Dynamic Geometry Software, starting with Cabri and Sketchpad in the early 90's, and its widespread adoption nowadays created a whole new area of research in mathematics education.

The mainly used scenarios are *demonstrations* that replace the printed figures in textbooks or hand-drawn figures on the blackboard, and *explorations* where students can interact with such figures in search for certain properties. In rare cases, students also *construct*, that is, they use the software as a modelling tool for their work.

There is no doubt, though not much scientific proof, that interactive figures can help students to understand a certain situation better. In fact, if designed carefully, these figures are able to give more information than static ones – however, more information does not necessarily mean easier understanding. On the other hand, the immediateness takes away hurdles in understanding that might be necessary to cross for a true mental progress. This corresponds to findings about the use of calculators that can cause a loss of qualitative understanding of functions (Quinn 2009). As mentioned, this remains a topic for investigation, and it is for sure that there are many factors that have to be considered if we really want interactive geometry to improve learning (cf. also Hoffkamp 2009).

One way to circumvent this problem – a missing connection between visualization and interaction on the one hand and understanding and internalization on the other – is to change the teaching approach to include activities of the students that helps them to construct knowledge and skills. This can be done, for example, by letting students work with interactive exercises (Kortenkamp & Richter-Gebert 1998, Kortenkamp 1999). Anyhow, the available material for doing constructions is limited, and already Klein (1968, p. 233) claims that "Triangle Geometry" and "mysterious points" do not convey an adequate mathematical content. This strongly indicates that there is a need for process-oriented teaching and learning methods that concentrate on activating the students.

Mathematics in school is not copying the usual way of doing mathematics professionally. One of the reasons is that a certain basis of skills is needed in order to work reasonably. A great variety of (mathematical) methods is necessary to work authentically. As a solution, it is possible to use an applied approach that is highly connected with experimental work in the sciences, using real data (again, this has been demanded earlier, most prominently for mathematics teaching by Klein). Students collect data and use interactive tools to analyze and interpret them. Students can rely on these tools to provide mathematical methods that they couldn't apply without them. To mention just one of these tools here we want to highlight Fathom (Key Curriculum, 2008), a software package that enables students to easily analyze collected data.

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In science teaching, experimentation is a crucial concept, in particular due to the methodological parallelism to "real" science. The "Nature of Science" concept has been in focus during the last years (e.g. Lederman, 1992; Höttecke, 2001; Bell et al. 2003, Schwartz et al. 2004), doing research on the conceptions of students about science and its influence to learning.

Although experimentation has a fairly long tradition in teaching the situation seems to be similar to the use of DGS in teaching scenarios. There is no evidence that real experimentation does indeed help to learn (Hofstein & Lunetta, 2004), but at least it seems to be possible to increase the learners' interest in sciences and positively influence their attitude towards it (Killermann, 1995; Klautke 1997). Duit (1989) summarizes that students' expectations even change their perception. A striking example is that students who are sure that electricity is being "used up" in a light bulb are sure that they see a small loss of current if measured before and after the bulb, although there is none. This underlines that the *complete* process of experimentation, including analysis and reflection, is necessary in teaching. Here again a mathematical treatment and modelling can help.

With regard to experimentation, it is also possible to use mathematical software to introduce the relatively new field of "experimental mathematics" (Borwein & Bailey 2008, see also http://www.experimentalmath.info) into teaching. Again, interactive tools can enable students to do this purely inner-mathematical work, provided they satisfy certain conditions on exactness and immediateness (Kortenkamp 2004).

Summarizing, the vital concepts in changing teaching from "visualization only" to "activate learning processes" seem to be the inclusion of real data and experimentation. There is an apparent connection between physics and other sciences, mathematics, experimentation, simulation and interactivity that is underlying the design of teaching and learning.

This article gives a theoretical framework that connects modelling, experimentation, and simulation. While we do not claim that any of these approaches improves learning per se, it is important that we are aware of how we use each component in the design of teaching/learning activities. Also, we point out the role of new software developments that enables students and teachers to make the transitions.

2   MODELING WITH REAL DATA

If real data is to be used in mathematics teaching, this can be acquired using various methods. If the objects of interest are in reach of the students, they can (and should) use measurement tools to collect the data themselves. Examples are height and weight of people, distances between places of interest, time intervals between events. Also, they can collect

information that needs no measurement, for example by counting or looking them up (numbers of people, interview answers, prices of various goods).

The Internet made it possible to easily acquire also data that has been measured (or counted, or specified) by others. Students can retrieve the population of cities and countries, tides of rivers and sees, weather data, poll answers, etc. This already introduces a "virtual component" to real data.

The mathematical treatment of data can be done using standard software such as EXCEL, the built-in spreadsheet applications in advanced calculators like the (TI n-spire or Casio Classpad), the spreadsheet add-ons in DGS like Cabri or GeoGebra, and many others; there even exists a specialized journal *Spreadsheets in Education* (http://epublications.bond.edu.au/ejsie/). Also, it is possible to just use any programming language supporting lists, which includes CAS. Depending on the type of exploration, a specialized tool for data analysis, in particular one that supports a student-friendly user experience like Fathom or its younger cousin Tinkerplots, can give a much better insight, even in elementary school (Biehler 2007).

Looking at software, this also reveals a hybrid approach to data acquisition. The Coach 6 software package (Heck et al. 2009, see also http://www.cma.science.uva.nl) provides ways to use digital images and even video recordings to measure and acquire data. Instead of using measurements coming from sensors (which is also possible), students can measure distances and, in the case of video, speed *on screen*. Image analysis and automated tracking support them in that activity. Heck and Hollstein (2002) describe how, for example, hanging bridges can be explored using the software.

We can summarize the ways of data acquisition in the diagram below.
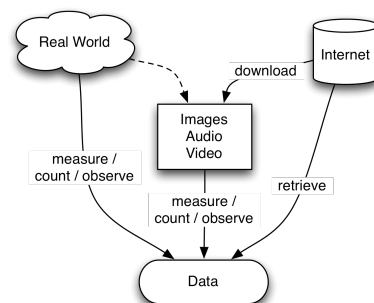
3   EXPERIMENTATION SCENARIOS

Experimentation is the process of controlled data acquisition by changing parameters in an experimental setup in a repeatable fashion. As such, it is intimately connected with the concepts described in the previous section. In this

and the following section two important additional factors of experimentation using computers will be examined: The *control mechanisms* that are available for experimentation in reality and the virtual world, as well as the underlying *models* that are being explored.

We start by listing several computer-based or computer–supported experimentation scenarios. The list is by no means complete, as there exists lots of packages for simulation and experimentation on the computer, but meant to show the essential differences in these approaches. Also, the list is biased to software that is used in mathematics education, apart from the first example.

3.1 Interactive Screen Experiments

Interactive Screen Experiments (ISE or IBE for German: Interaktive Bildschirmexperimente) were developed for schools and deliver experiments that are too dangerous, expensive, or time-consuming in that environment via series of digital images (Kirstein & Nordmeyer, 2007). Students can vary parameters on screen, for example the voltage used, and observe the effects of these changes. The screen representations use digital images of the real experiment that have been taken in the universities' laboratories before. Students can only see the effects of actions that have been recorded in advance. They can only deviate from the experimental setup if this has been included in the ISE by the authors.
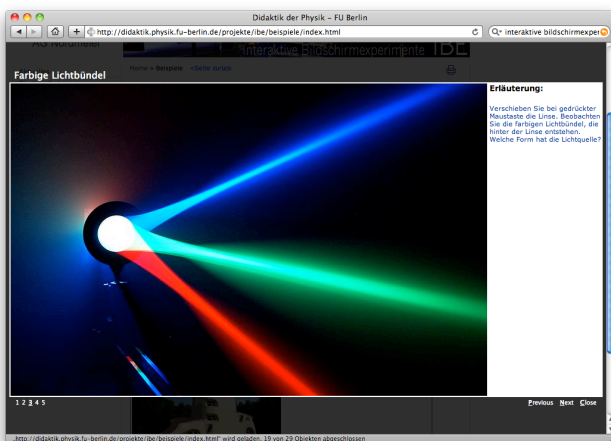

Figure 1: Screenshot from ISE. Students can control the distance of the lens from the light source to observe changes.

3.2 PHUN physics simulation

The PHUN package by Emil Ernerfeldt[1] is a microworld for physics simulation. It is possible to create

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[1] PHUN (http://www.phunland.com) is now part of algodoo (http://www.algodoo.com) by Algoryx Simulation, see http://algoryx.se.

physical objects (solids, hinges, gears, chains, springs) of various materials (differing in density, elasticity, etc.) in 2D. The system then simulates the physical interaction between these objects. It is possible to create fairly complex setups that would be difficult or impossible to realize in the real world. During the simulation the user can interact by adding further elements or moving existing ones. It is not possible to measure forces or other data during the simulation.

3.2 Coach 6

We already mentioned Coach 6 in the process of data acquisition. The software can also be used to create simulations using a visual modeling language based on mathematical formalization of a problem. That is, it is possible to compare the data acquired from a video of a spring pendulum, e.g., to the idealized version of the same experiment using a damped sine function (or an equivalent differential equation). Also Coach 6 supports real-time input from various – also custom – sensors, and output to actuators like motors, heat elements or others.
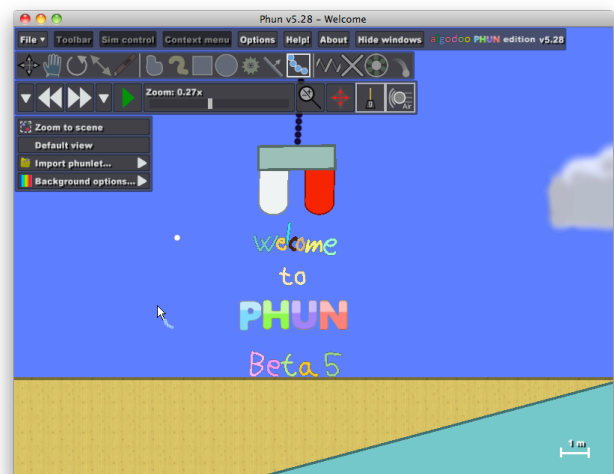

Figure 2: Screenshot of PHUN

3.3 Cinderella

Cinderella (Richter-Gebert & Kortenkamp 1998, 2006) was originally created as a "software for doing geometry on a computer". With the release of version 2.0 a physics simulation engine and a scripting language (see Richter-Gebert & Kortenkamp 2009) have been introduced that enable users to easily work with experimental setups. For example, it is easily possible to explore the first Keplerian law of planet motion by "constructing" (i.e. drawing) a planet and a sun as shown in Fig. 2. Using the scripting language, the data from the simulation can be retrieved and analyzed.
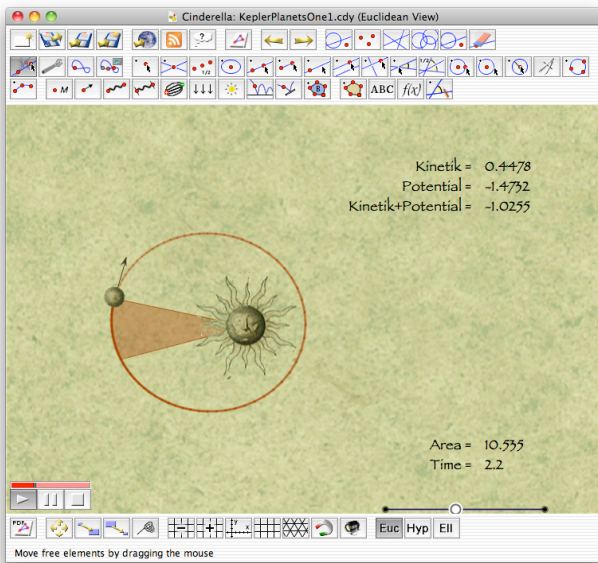
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Figure 3: Exploring Keplers' law in Cinderella.

A more sophisticated example explores the geometry of hanging bridges (Fig. 4). In contrast to the approach described by Heck and Hollstein (2002) the physics simulation engine is used to create an ideal bridge and the geometry part is used to project that simulation onto a digital image of the bridge. The "ideal bridge" uses rubber-bands to connect the weights (represented by point-like masses).
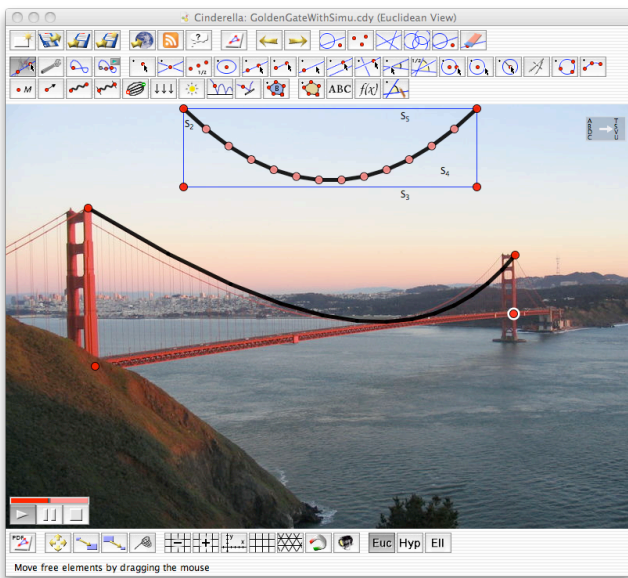

Figure 4: The Golden Gate Bridge and a hanging bridge simulation in Cinderella. A video is available on YouTube at http://youtube.com/watch?v=xHY6G2fTLPg.

When the simulation is started, the chain of rubberbands and masses automatically takes the shape in a parabolic arc. By varying the ratio of masses and gravity the simulation can be

adjusted to see that it corresponds to the original bridge. More details can be found in Kortenkamp & Richter-Gebert (2008).

It is also possible to connect geometric constructions and simulations to external sensors and actors via CindyScript. The language supports LEGO MINDSTORMS NXT™ via Bluetooth as well as the acceleration sensors found in Apple Laptops. An example of the interplay between mathematical software, sensors and actors can be seen in a video demonstration that is available at http://youtube.com/watch?v=I8VvTENzPGI. The sensor data from the acceleration sensor is used to control the two motors of a small LEGO car (Fig. 5). It is necessary for the students to find a suitable transformation from the 3D acceleration vector to the pair of speeds for the motors.
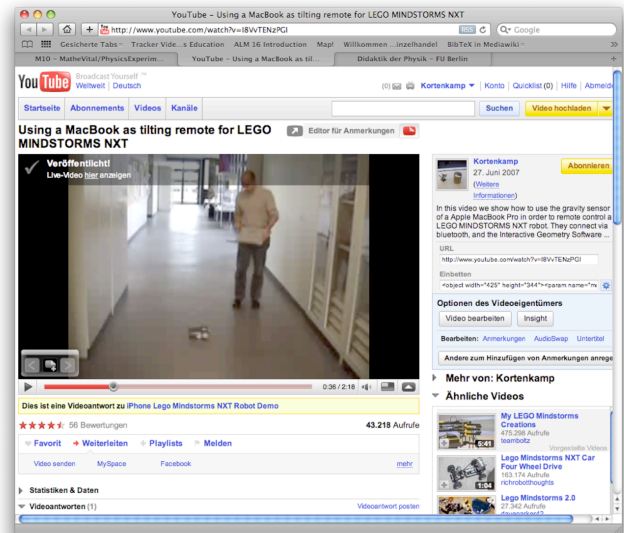

Figure 5: Video on YouTube showing the connection between acceleration sensor and LEGO robot.

4   ANALYSIS

In addition to the data acquisition methods from Sec. 2 and based on the examples above we can identify several dimensions that can be used to classify (or at least compare) experimental setups that use computers or, more generally, technology. We also try to classify the example software shown above, however, these examples usually are not software-specific, but software-use specific, and thus depend on the design of the learning activity.

(i)   The experimental design and setup can be prescribed (ISE) or done freely by the student or teacher (PHUN, Coach 6, Cinderella).

(ii)   The control parameters can be varied in certain ranges only (ISE) or may be changed also to values that were not foreseen by the designer of the activity (PHUN, Coach 6, Cinderella).

(iii) The on-screen representation of the experiment may be abstract (PHUN) or close to reality (ISE), or both (Coach 6, Cinderella).

(iv) Design of an experiment, unless prescribed, is done symbolically using a specification language (Coach 6) or implicitly based on a microworld metaphor (PHUN), or both as a mixture (Cinderella).

(v) The underlying mathematical model is unknown and given by the real world (ISE) or based on an idealized built-in description using differential equations (PHUN, Cinderella), or both (Coach 6).

(vi) Data can be recorded and re-used for further analysis (Coach 6, to a lesser extend Cinderella, ISE) or is not available in symbolic form (PHUN).

(vii) Data from external sensors can be used to control the experiment (Coach 6, Cinderella)

(viii) The software can control actuators like motors using data from the experiment (Coach 6, Cinderella)

Apparently, *model* and *control* are important concepts for experimentation. All experiments share the assumption that there is a model that could be found, verified, or explained using the experiment. Also, all experiments need some kind of controlled variation to give any results. This leads us finally to a classification scheme for blended experimentation. We cannot define levels of blending, as the different blending components are independent, but we can define blending types that are non-exclusive apart from Type 0.

TYPE 0: There is no connection between the real experiment and the virtual experiment, or only one type, either real or virtual, is used

TYPE M: Virtual experiment and reality are connected via a shared mathematical model

TYPE S: A virtual experiment is using sensor data from reality

TYPE A: A real experiment is controlled using actuators

5   CONCLUSION

Experimentation is an important concept not only in the natural sciences, but also in mathematics. We identified ways of acquiring data and eight dimensions of experimental design that can be used to classify computer-supported or computer-based experiments. In addition to purely virtual or purely real experiments it is also possible to blend these two approaches. By blending, it might be possible to preserve the motivational aspects, the positive attitude, the scientific concept, and the credibility of real experiments, while still enabling students to work with experimental setups that would not be possible to be done in the classroom otherwise. Our classification scheme can be used as a basis for the design of such activities.

On the software side, there are several software packages that can support blended experimentation. We mention two of them, Coach 6 and Cinderella, both originating in different mathematical software concepts.

Another aspect that we did not cover in this article is the suitability of virtual experimentation software for teaching based on its mathematical accuracy. For example, the numerical stability of the integrator engine that finds solutions for the differential equations underlying the simulation is crucial for the correctness of the effects that students can observe. For example, by moving from a simple Euler-integrator to a more advanced Runge-Kutta-Type integrator like Dormand-Prince45 in Cinderella we could greatly improve the reliability and correctness of virtual experiments. See also Kortenkamp (2004) for a discussion of the suitability of mathematical software for experimentation.

So far there is no empirical evidence that blended experimentation approaches can help for learning. Suitable scenarios are still to be identified and then need further inspection. By using the design dimensions provided in this article it might be easier to identify the underlying design principles for such activities. The graduate school "Mathematisch-naturwissenschaftliches Lernen in lebensnahen Anwendungskontexten" (Learning of mathematics and sciences in application contexts near to life), located at the Universities of Education Ludwigsburg, Schwäbisch Gmünd and Weingarten, Germany, is currently investigating these issues on a scientific level, and we expect first results in 2012. Still, this is just a start and we consider it worthwhile to do more research in that direction.

As a final remark we want to mention the apparent connection of Computer Games, in particular first-person 3D games, to experimentation. Again, the player is in control of various parameters (direction, speed…) and can observe the results of changing these parameters. Using new human interaction devices the connection between the real world and the virtual world will probably become much closer in the not-so-far future. Apart from a superior gaming experience, we should explore the implications for education and learning. Blended experimentation can be a starting point here as well.

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BIOGRAPHICAL NOTES

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Jürgen Richter-Gebert is full professor for "Geometry and Visualization" at the Technical University Munich, Germany. His special interests include automatic theorem proving, dynamic geometry, geometric programming, and many more.

Both authors developed the geometry software Cinderella, whose first version was available in 1999. The software received many awards, among them the European Academic Software Award and the Deutsche Bildungssoftwarepreis.